On prescribing CG convergence

Gérard Meurant[1]

[1] Paris, France

Abstract

In this talk we show how to construct real symmetric positive definite square matrices A of order n and real right-hand sides b for which the Conjugate Gradient (CG) algorithm of M. Hestenes and E. Stiefel [3] applied to $Ax = b$ has a prescribed residual norm convergence curve. For a description of the CG algorithm and a study of its properties, we refer to [4]. We also consider prescribing as well the A-norms of the error.

The Full Orthogonalization Method (FOM) of Y. Saad [5] for solving nonsymmetric linear systems is equivalent to CG when the matrix is symmetric. Prescribing the residual norms for FOM was studied some years ago. One can construct linear systems for which the matrix has prescribed eigenvalues and such that FOM delivers prescribed residual norms and also prescribed Ritz values at all iterations; see [1].

We are interested in discussing what can be done for CG. We will construct symmetric tridiagonal matrices T such that CG yields prescribed relative residual norms with the right-hand side e_1, the first column of the identity matrix. Then, the linear system $Ax = b$ is obtained from $A = VTV^T$ and $b = Ve_1$ where V is any orthonormal matrix. Moreover, it turns out that in our construction there are some free parameters that can be chosen to prescribe as well the A-norms of the error.

We consider two ways of constructing T. The first one uses results from A. Greenbaum and Z. Strakoš [2] to construct T^{-1}. The second one is a more direct construction of the non-zero entries of T. Then, we illustrate the results with some numerical examples.

References

[1] J. Duintjer Tebbens and G. Meurant, *Any Ritz value behavior is possible for Arnoldi and for GMRES*, SIAM J. Matrix Anal. Appl., v 33, n 3 (2012), pp. 958–978.

[2] A. Greenbaum and Z. Strakoš, *Matrices that generate the same Krylov residual spaces*, in Recent advances in iterative methods, G.H. Golub, A. Greenbaum and M. Luskin Eds., Springer (1994), pp. 95–118.

[3] M.R. Hestenes and E. Stiefel, *Methods of conjugate gradients for solving linear systems*, J. Nat. Bur. Standards, v 49 n 6 (1952), pp. 409–436.

[4] G. Meurant, *The Lanczos and conjugate gradient algorithms, from theory to finite precision computations*, SIAM, Philadelphia (2006).

[5] Y. Saad, *Krylov subspace methods for solving large nonsymmetric linear systems*, Math. Comp., v 37 (1981), pp. 105–126.