Fast and stable estimates for matrix functionals

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Abstract

The approximation of matrix functionals appears in many applications arising from the fields of mathematics, statistics, mechanics, networks, machine learning and physics. In this paper, we estimate matrix functionals of the form $X^T f(A)Y$, where $A \in \mathbb{R}^{p\times p}$ is a given diagonalizable matrix, $X, Y \in \mathbb{R}^{p\times k}$ are skinny "block vectors" with $k \ll p$ columns and f is a smooth function defined on the spectrum of the matrix A. We apply a direct backward stable approach based on the extrapolation of the moments of the given matrix, for estimating this kind of matrix functionals. This approach avoids the application of the polarization identity, relies only on inner products and matrix vector multiplications, and therefore, the computational speed for large p is quite remarkable. Specific applications from Statistics and Network Analysis are presented, for which the extrapolation estimates are highly recommended.

References

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